A Periodic Structure of Cylindrical Posts in a Rectangular Waveguide*

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Summary—The propagation characteristics of a rectangular waveguide loaded with uniformly spaced cylindrical posts (periodic structure) are investigated at a frequency of 2840 Mc. A qualitative discussion on the expected behavior of the effective guide wavelength of this type of periodic structure is presented, and it is shown that the presence of the posts reduces the guide wavelength of the waveguide. The guide wavelength is then measured as a function of post diameter, post depth, and post spacing; and curves enabling one to design periodic structures which have guide wavelengths in the region of the free space wavelength are presented.

Introduction

THE propagation characteristics of a rectangular waveguide are modified when the guide is loaded with uniformly spaced posts. The purpose of this investigation is to determine how the effective guide wavelength of the periodic structure shown in Fig. 1 varies as a function of post geometry and its depth of penetration.

To understand the behavior of this periodic structure it is first necessary to examine the equivalent circuit of a single element of the structure. A cylindrical post of variable height and diameter in a rectangular waveguide can be represented by the four-terminal network shown in Fig. 2. An approximate theoretical solution for this post has been obtained by Suzuki;[1] however, the final expressions for the reactances are very difficult to evaluate. Experimental results have been obtained by Marcuvitz[2] and although they cannot be scaled directly to the waveguide and post dimensions being investigated, they can be used to describe the qualitative behavior of the post well enough to analyze the periodic structure.

In Fig. 2 jx_b is a relatively small series reactance that increases slightly with post depth and diameter. The main effect of the post is to introduce a shunt reactance jx_a across the waveguide. In Fig. 3 jx_a is plotted as a function of post depth h/b for various post diameters d. As the post penetration is increased, the capacitive reactance of the post approaches zero (when

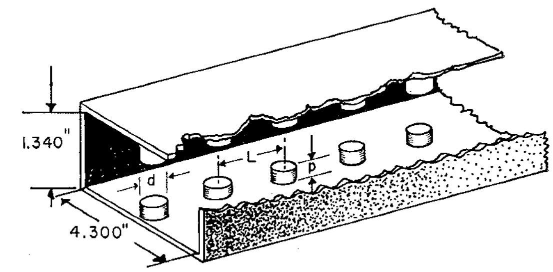

Fig. 1—Periodic structure.

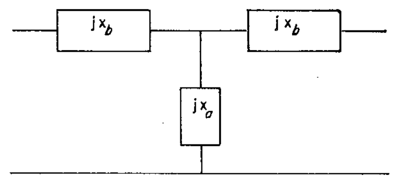

Fig. 2—Equivalent circuit of post.

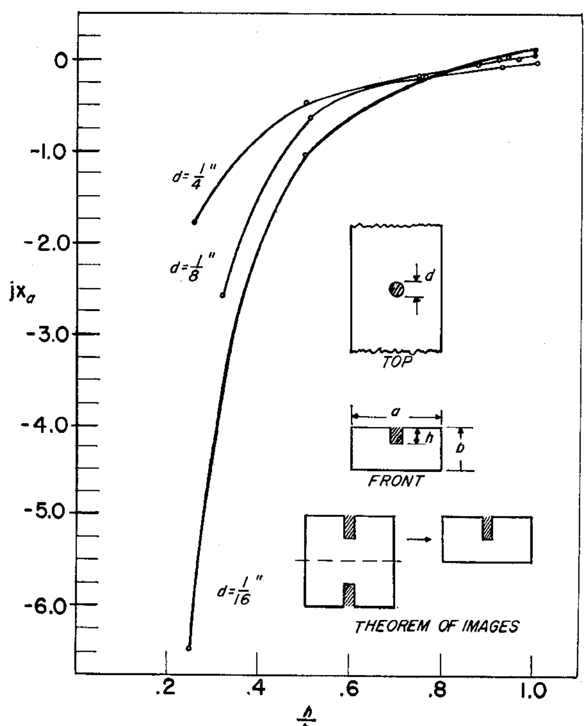

Fig. 3—Shunt reactance of post. (*Experimental curves from Marcuvitz.*[2])

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[1] M. Suzuki, "Circuit Parameters of a Tuning Post in a Rectangular Waveguide and its Applications," AF Cambridge Research Center, Bedford, Mass., AFCRC-TN-57-764 [Rept. R-591-57, PIB, 519, AFCRC Contract No. AF 19(604)-2031]; July, 1957.

[2] N. Marcuvitz, "Waveguide Handbook," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 10, p. 271; 1947.

it behaves as a short circuit) and then becomes inductive. It can be shown from the theorem of images (see Fig. 3) that the shunt reactance of the double post being investigated is equivalent to that of a single post in a waveguide of half the height.

The effective guide wavelength λ_g' and characteristic impedance Z' of a periodic structure are related to the guide wavelength λ_g and characteristic impedance Z of the unloaded waveguide as follows:

$$\cos \frac{2\pi}{\lambda_g'} L = \cos \frac{2\pi}{\lambda_g} L - \frac{1}{2x_a} \sin \frac{2\pi}{\lambda_g} L, \quad (1)$$

$$\frac{Z'}{Z} = \frac{\tan \frac{\pi}{\lambda_g} L}{\tan \frac{\pi}{\lambda_g'} L}, \quad (2)$$

where L is the distance between posts, and x_a is the normalized shunt reactance of the posts.[3] (It is assumed that $x_b = 0$.)

Eqs. (1) and (2) are valid only when the posts are far enough apart so that the coupling between them is negligible. At a fixed frequency (λ_g=constant), λ_g' is a function of x_a and L. For a fixed post separation of L/λ_g, as x_a is made less negative, $\cos(2\pi L/\lambda_g')$ approaches -1. When $\cos(2\pi L/\lambda_g')$ is greater than -1, it becomes indeterminant and this corresponds to a cut-off region in the waveguide. For a fixed L/λ_g, therefore, λ_g' can be decreased by only a limited amount regardless how small x_a is made. Upon examining the limiting condition where

$$\cos \frac{2\pi L}{\lambda_g'} = -1,$$

however, it can be seen that $\lambda_g' = 2L$ and so λ_g' can be made smaller by decreasing the distance between posts. When this distance becomes too small, the coupling between posts cannot be neglected, (1) is no longer valid, and λ_g' cannot be further decreased.

When x_a is made inductive (positive), $\cos(2\pi L/\lambda_g')$ approaches $+1$ and λ_g' approaches infinity. If the posts become too inductive

$$[\cos(2\pi L/\lambda_g') > 1],$$

the TE_{10} mode does not propagate.

The behavior of the periodic structure of cylindrical posts can be summarized as follows: When the post penetration is small the post can be represented by a shunt capacitance, and the guide wavelength λ_g' of the periodic structure is less than the guide wavelength λ_g of the unloaded waveguide. As the post penetration is further increased, λ_g' is further decreased until a cut-off region is reached and propagation of the TE_{10} mode ceases. With a further increase in post penetration, the shunt reactance passes through a resonance ($x_a = 0$) and then becomes inductive. When the inductive reactance increases to a large enough value such that $\cos(2\pi L/\lambda_g') < +1$, propagation resumes and λ_g' decreases from infinity down to a value larger than λ_g as the post is extended all the way across the waveguide.

[3] J. Brown, "The design of metallic delay dielectrics," *Proc. IEE*, vol. 97, pt. 3, p. 45; January, 1950.

Experimental Procedure

The guide wavelength λ_g' of a periodic structure of known length can be determined by measuring the phase of a traveling wave at the beginning and end of the structure if the guide wavelength λ_g of the unloaded waveguide is known. The phase measurement is made using the experimental setup shown in Fig. 4. The input signal is divided into two parts; one part travels through the periodic structure under test, and the other part travels through a slotted line that is used as a reference phase line. To obtain precise data it is necessary that both signals be traveling waves. Each waveguide is therefore terminated in a matched load. The input signals to arms 1 and 2 of the magic T are adjusted so that they are approximately equal in amplitude. A sharp null will appear at output arm 3 when the input signals are 180° out of phase and arm 4 is terminated in a matched load. The phase difference between two signals is determined from the distance that the probe on the phase line (slotted line No. 1) has to be moved to produce a null at output arm 3.

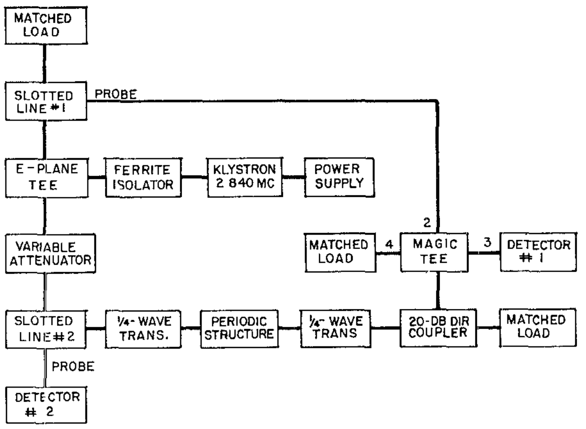

Fig. 4—Block diagram of equipment.

The periodic structure consists of a uniform section of waveguide with adjustable posts and behaves as a uniform waveguide with a propagation constant and characteristic impedance different from that of the unloaded waveguide. The phase of the signal from the directional coupler at the end of the periodic structure is first measured with the posts at zero depth. The post penetration is then slightly increased and the measurement is repeated. The periodic structure has an

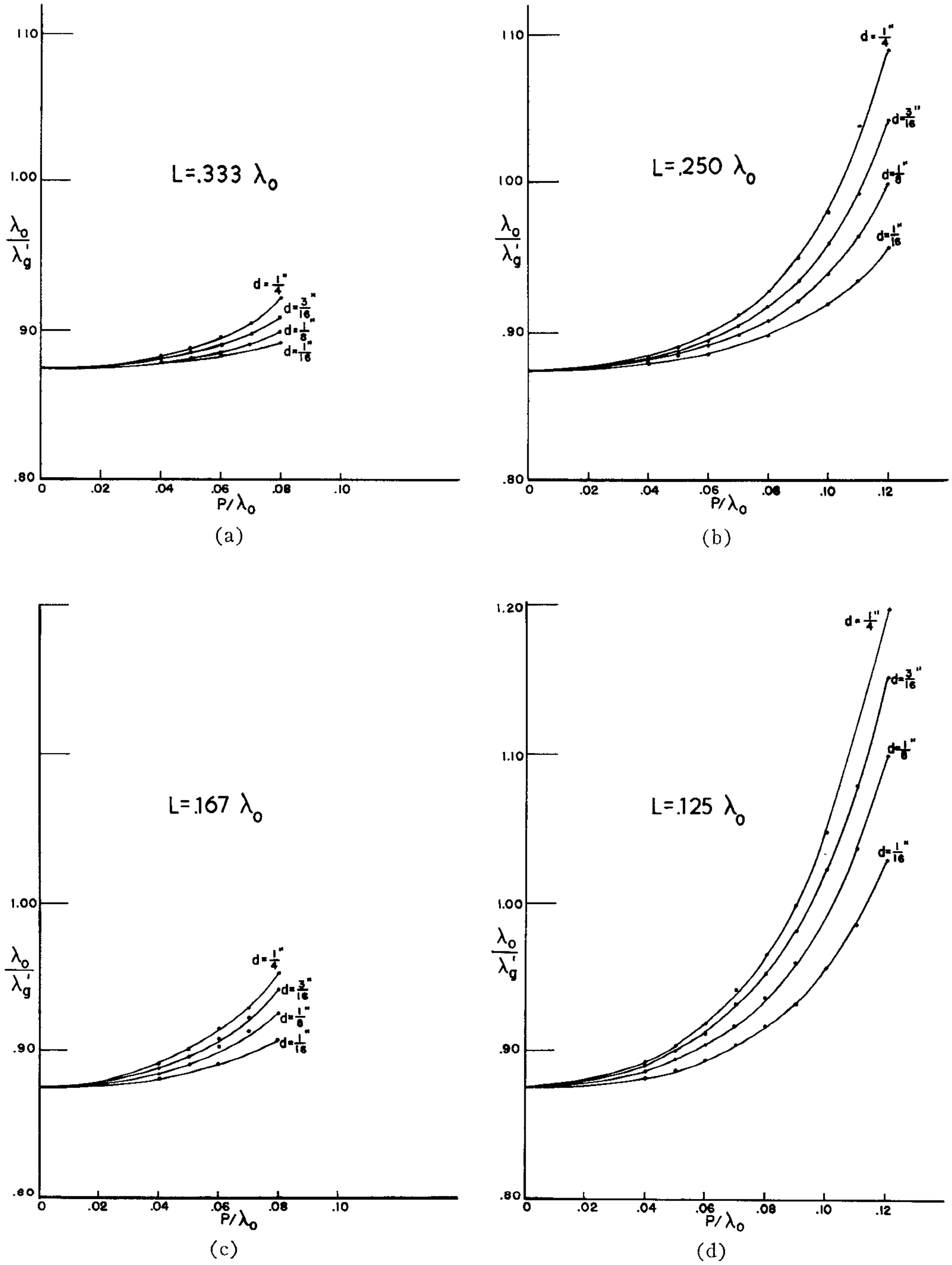

Fig. 5—Guide wavelength vs post depth for constant post spacing.

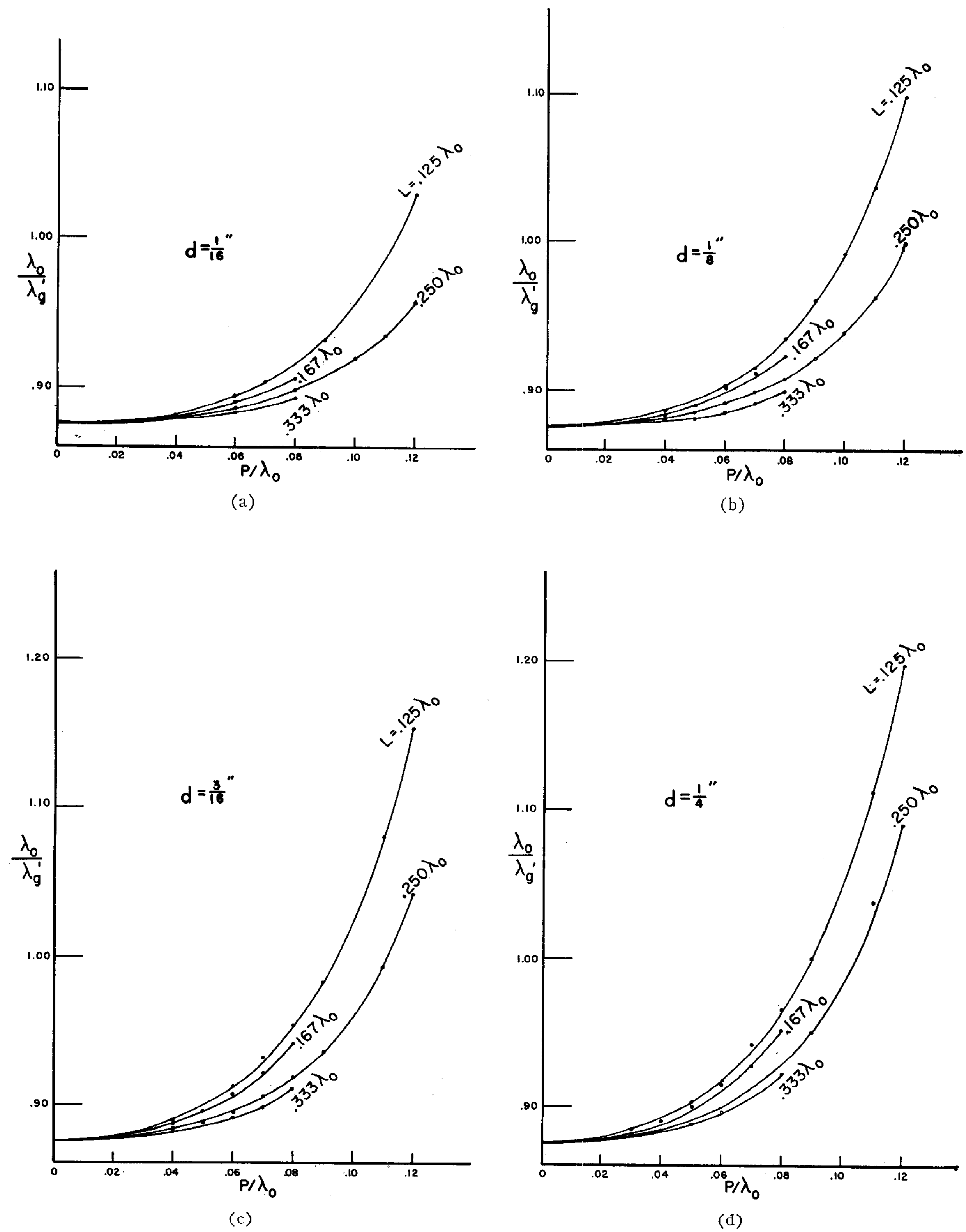

Fig. 6—Guide wavelength vs post depth for constant post diameter.

electrical length of 630° ($1.75\lambda_g$) with zero post penetration. As the post depth is increased, the shift in phase increases. Since this corresponds to an increase in the electrical length of the structure, the guide wavelength is decreased. The ratio of freespace wavelength λ_0 to the guide wavelength λ_g of the unloaded structure is 0.875. The ratio of λ_0 to λ_g' is

$$\frac{\lambda_0}{\lambda_g'} = 0.875\left[1 + \frac{\Delta\theta}{630°}\right],$$

where $\Delta\theta$ is the change in phase in degrees due to the presence of the posts.

Measurement of the input VSWR of the periodic structure checks the relationship.

$$\frac{Z'}{Z} = \frac{\tan\frac{\pi}{\lambda_g}L}{\tan\frac{\pi}{\lambda_g'}L}.$$

This is possible because the input VSWR is composed of reflections from the interface from Z to Z' and the interface from Z' to Z. Since the characteristic impedances are assumed real, the VSWR that is due to each interface is R/R' for $R>R'$. The resultant input VSWR can vary from a minimum of 1.0 to a maximum of $(R/R')^2$, depending on the electrical distance between interfaces. It is 1.0 when the interfaces are an even number of quarter-wavelengths apart and $(R/R')^2$ when they are an odd number of quarter-wavelengths apart. Due to the reflections at the interfaces, there is a slight difference between the phase angle of the transmission coefficient that is measured and the electrical length of the periodic section. Since the reflection coefficients are small, the error introduced is negligible.

Discussion of Results

The guide wavelength λ_g' of the periodic structure shown in Fig. 1 was measured as a function of post diameter d, penetration p, and spacing L, at a frequency of 2840 Mc ($\lambda_0=4.159$ inches). In Figs. 5 and 6, λ_0/λ_g' is plotted as a function of p/λ_0. Each set of curves in Fig. 5 corresponds to a fixed post spacing L. Each set of curves in Fig. 6 corresponds to a fixed post diameter d. It can be seen that λ_0/λ_g' increases slowly with an increase in post diameter and depth and a decrease in post spacing. These results agree very well with those expected from the qualitative discussion that was presented earlier. Since this investigation was conducted primarily in the region where $\lambda_g' \cong \lambda_0$, the actual cutoff conditions of the periodic structure were not established; but the fact that the slopes of the curves become very steep for large post diameter and penetration indicates that a cutoff condition is rapidly being approached.

The results of the input VSWR measurements were found to be consistent with those expected from (2). The assumption that this periodic structure can be treated as a uniform waveguide with a real characteristic impedance is therefore valid.

Acknowledgment

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